

# The quadrupole moments of the $11^-$ isomers in $^{194,196}\text{Pb}$ revisited

K. Vyvey<sup>1</sup>, D.L. Balabanski<sup>2,3</sup>, and G. Neyens<sup>1</sup>

<sup>1</sup> Instituut voor Kern- en Stralingsfysica, University of Leuven, Celestijnenlaan 200 D, B-3001 Leuven, Belgium

<sup>2</sup> Dipartimento di Fisica, Università degli Studi di Camerino, 62032 Camerino (MC), Italy

<sup>3</sup> Institute for Nuclear Research and Nuclear Energy, Bulgarian Academy of Sciences, BG-1784 Sofia, Bulgaria

Received: 5 November 2004 /

Published online: 23 December 2004 – © Società Italiana di Fisica / Springer-Verlag 2004

Communicated by D. Guereau

**Abstract.** The measured spectroscopic quadrupole moments and deduced deformations of the  $^{194,196}\text{Pb}(11^-)$  intruder isomers are reevaluated, making use of the recently measured magnetic moments of these states. This adjustment is necessary since the quadrupole moments were obtained from Level Mixing Spectroscopy measurements, being sensitive to the ratio of the quadrupole interaction frequency and the magnetic moment. The results confirm the moderate deformation of the  $\pi(3s_{1/2}^{-2}1h_{9/2}1i_{13/2})_{11^-}$  states. A systematic comparison between the updated experimental and the available theoretical values is presented.

**PACS.** 21.10.Ky Electromagnetic moments – 21.60.-n Nuclear structure models and methods – 27.80.+w  $190 \leq A \leq 219$

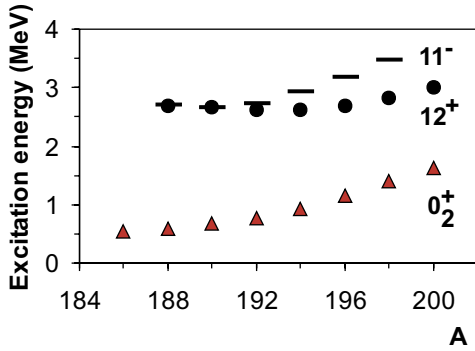
## 1 Introduction

Many nuclear structure phenomena manifest themselves in the light Pb isotopes, varying from shape coexistence between spherical, prolate, oblate and, sometimes, superdeformed states to excitations associated with the shears mechanism. Until today, these observed structures are subject of intensive theoretical and experimental studies [1, 2, 3, 4, 5, 6]. In this note we will focus on the deformation of  $^{194,196}\text{Pb}(11^-)$  intruder isomers. We summarize all experimental values necessary to deduce the spectroscopic quadrupole moments of these states and compare the re-evaluated experimental values for the quadrupole moments with several theoretical calculations. This review is needed because of the new measurements of the  $g$ -factors of the  $11^-$  intruder states [7]. The quadrupole moments were obtained from Level Mixing Spectroscopy measurements, being sensitive to the ratio of the quadrupole interaction frequency,  $\nu_Q$ , and the magnetic moment,  $\mu$ . Therefore, precise knowledge of the  $g$ -factors is necessary for a reliable deduction of the quadrupole moments.

The  $11^-$  proton excitations out of the  $Z = 82$  core are of particular interest for two reasons. On the one hand, it is widely accepted that the  $11^-$  states are shape isomers whose character can only be explained by assuming an oblate deformation. On the other hand, the  $11^-$  isomers have the same configuration as the proton blade in

a shears band. The spins and lifetimes of the  $11^-$  states allow to study both their single-particle structure and deformation by measuring their static electromagnetic moments. Nuclear moment measurements are not possible for the observed oblate  $0^+$  states and the quasi-prompt shears bandheads (with the exception of the  $^{193}\text{Pb}(29/2^-)$  state [8, 9]). Therefore, experimental values of the quadrupole and magnetic moments of the  $11^-$  isomers provide additional information on both the nuclear excitations within the oblate potential well and the shears excitations.

To date, the  $11^-$  states have been observed from  $^{188}\text{Pb}$  to  $^{198}\text{Pb}$  [10, 11, 12, 13, 14, 15] (fig. 1). They are interpreted as two-quasiparticle excitations across the  $Z = 82$  shell gap, involving a  $\pi(3s_{1/2}^{-2}1h_{9/2}1i_{13/2})$  single-particle configuration. Experimental evidence originates from  $g$ -factor measurements in  $^{194,196}\text{Pb}$  [7, 16]. Both, theoretical calculations and experimental fingerprints indicate that the  $11^-$  isomers must take an oblate shape. Their low excitation energies, *e.g.*, are explained by a combination of enhanced pairing interactions and a large energy gain associated with an oblate deformation, making the  $1h_{9/2}$  and  $1i_{13/2}$  orbitals intrude into the low-energy part of the spectrum. More recently, surprisingly enhanced  $E3$  strengths have been observed for the ( $11^- \rightarrow 8^+$ ) transitions in the even neutron-deficient Pb isotopes ( $N \leq 114$ ). The effect has been linked with configuration mixing in the  $8^+$  state when oblate deformation is established [17]. Often, information on the nuclear deformation is extracted from



**Fig. 1.** Energy systematics of the oblate  $11^-$  (bars) and spherical  $12^+$  (circles) isomers in the neutron-deficient Pb-isotopes [10, 11, 12, 13, 14, 15]. For comparison the systematics of the oblate  $0^+$  states (triangles) is added to the picture as well [19, 20, 21, 22, 23].

the properties of the rotational bands built on top of the state of interest. For the  $11^-$  excitations, though, only a few indications for rotational bands have been observed, all of them showing irregularities [11, 18].

## 2 Re-evaluation of the experimentally deduced quadrupole moments

More direct evidence for a moderately deformed shape came from the measurement of the spectroscopic quadrupole moments of the  $11^-$  isomers in  $^{194,196}\text{Pb}$  [24, 25]. They have been measured at the CYCLONE facility in Louvain-la-Neuve by applying the LEMS technique [26], being sensitive to the ratio  $\nu_Q/\mu$ . The quadrupole interaction frequencies for the  $^{196,194}\text{Pb}(11^-)$  isomers implanted in a Re crystal, as reported in refs. [24, 25], were deduced by using the older experimental  $g$ -factor of the  $11^-$  state in  $^{196}\text{Pb}$ :  $g = 0.96(8)$  [16]. By assuming that  $g(^{194}\text{Pb}, 11^-) = g(^{196}\text{Pb}, 11^-)$ , the quadrupole frequencies of  $\nu_Q(^{196}\text{Pb}, 11^-) = 199(32)$  MHz [24] and  $\nu_Q(^{194}\text{Pb}, 11^-) = 262(41)$  MHz [25] were extracted.

Recently, the  $g$ -factors of both the  $^{196}\text{Pb}(11^-)$  and  $^{194}\text{Pb}(11^-)$  isomers have been measured at the ANU 14 UD pelletron, resulting in the more accurate values of  $g(^{196}\text{Pb}; 11^-) = 1.04(5)$  and  $g(^{194}\text{Pb}; 11^-) = 1.03(2)$  [7]. The shift of  $\simeq 10\%$  in the  $g$ -factors induces a similar shift in the quadrupole interaction frequencies as extracted from the LEMS-data. The updated value for the quadrupole interaction frequency of the  $^{196}\text{Pb}(11^-)$  isomer, using the weighted mean of its magnetic moment values, is  $\nu_Q(^{196}\text{Pb}, 11^-) = 211(30)$  MHz. With the magnetic moment value for  $^{194}\text{Pb}(11^-)$ , we obtain  $\nu_Q(^{194}\text{Pb}, 11^-) = 281(38)$  MHz.

The electric field gradient of Pb in Re was determined from the ratios  $\nu_Q/\mu$  of the  $^{194,196}\text{Pb}(12^+)$  isomers in Re. For the magnetic moments of the  $12^+$  states, we adopt the weighted mean of the different experimental values in ref. [27]:  $\mu(^{194}\text{Pb}, 12^+) = -2.06(3)$  and  $\mu(^{196}\text{Pb}, 12^+) = -1.92(2)$ . The uncertainty is calculated as the

maximum of the uncertainty on the weighted mean and the weighted standard deviation. The value for the magnetic moment of the  $^{194}\text{Pb}$  isomer obtained in this manner, and hence also the extracted quadrupole frequency,  $\nu_Q$ , differs slightly from the one used in ref. [25], where the value of Stenzel *et al.* [28] was adopted. By making use of the experimental quadrupole moments of refs. [28, 29], we finally get  $|V_{ZZ}(^{196}\text{PbRe})| = 2.42(27) \cdot 10^{21}$  V/m<sup>2</sup> and  $|V_{ZZ}(^{194}\text{PbRe})| = 2.42(23) \cdot 10^{21}$  V/m<sup>2</sup> [24, 25]. Since the electric field gradient is isotope independent and these two values can be considered as two independent measurements, we adopt the value of  $|V_{ZZ}(\text{PbRe})| = 2.42(17) \cdot 10^{21}$  V/m<sup>2</sup>. The excellent agreement between the two independently deduced electric field gradients supports the suggested analysis procedures. In particular, the double perturbation analysis including the partial feeding of the  $12^+$  isomer via the  $11^-$  isomer is indeed necessary to determine the quadrupole frequencies of the  $11^-$  and the  $12^+$  isomers in  $^{194}\text{Pb}$ . The extracted electric field gradients differ with 25% if the feeding via the  $11^-$  isomer is not considered in the analysis [25]. In addition, both the quadrupole frequencies of the  $12^+$  isomers and the  $11^-$  isomers were extracted from the same set of data, allowing to deduce the quadrupole moments of the  $11^-$  isomers from the ratios  $\nu_Q(11^-)/\nu_Q(12^+)$ . Hence, it is excluded that systematic errors are present in the determination of  $Q_s(11^-)$ .

As a result, we can calculate the spectroscopic quadrupole moments from the frequency,  $\nu_Q = \frac{eQ_s V_{ZZ}}{h}$ , as  $Q_s(^{196}\text{Pb}, 11^-) = (-)3.6(6)$  b and  $Q_s(^{194}\text{Pb}, 11^-) = (-)4.8(7)$  b. LEMS measurements are not sensitive to the sign of the quadrupole moment. However, since theoretical models predict an oblate shape, we adopt the negative sign. A summary of the relevant values for the extraction of  $Q_s(11^-)$  and a comparison between the newly and formerly deduced values for  $Q_s$  is given in table 1.

## 3 Discussion

It is interesting to compare the updated values for the quadrupole moments with several theoretical calculations available in literature. Theoretical papers, though, often quote nuclear deformations,  $\beta_2$ , instead of spectroscopic quadrupole moments. The relationship between the spectroscopic quadrupole moment and the nuclear deformation is only straightforward for axially symmetric charge distributions with their spins aligned along their deformation axes (*i.e.*,  $K$  is a good quantum number and  $K = I$ ). Then we may use the formulae [30]  $Q_s = Q_0 \frac{3K^2 - I(I+1)}{(I+1)(2I+1)}$  and  $Q_0 = \frac{3}{\sqrt{5\pi}} R_0^2 Z \beta_2 (1 + 0.36\beta_2)$  in which  $Q_0$  is the intrinsic quadrupole moment and  $R_0 = r_0 A^{1/3}$  with  $r_0 = 1.2$  fm. Assuming that the former conditions are fulfilled we find  $Q_0(^{196}\text{Pb}, 11^-) = (-)4.7(7)$  b,  $Q_0(^{194}\text{Pb}, 11^-) = (-)6.2(1.0)$  b and  $\beta_2(^{196}\text{Pb}, 11^-) = (-)0.17(3)$ ,  $\beta_2(^{194}\text{Pb}, 11^-) = (-)0.23(3)$ . Note that the lack of well-established rotational bands on top of the  $11^-$  isomers might indicate that  $K$  is not a good quantum number, though another explanation might be that the bands have not been observed because they are not yrast.

**Table 1.** Overview of the experimental quantities necessary to extract  $Q_s(11^-)$  plus a comparison of the updated and formerly deduced quadrupole moments and nuclear deformations of the  $11^-$  isomers (see notations in the text).

Physical quantity	Unit	$^{194}\text{Pb}$	Ref.	$^{196}\text{Pb}$	Ref.
$ \nu_Q/\mu(12^+) $	$[\text{b}/\mu_N]$	14(1)	[25]	20(2)	[24]
$\mu(12^+)$	$[\mu_N]$	$-2.06(3)^{(a)}$	[27]	$-1.92(2)^{(a)}$	[27]
$ \nu_Q(12^+) $	$[\text{MHz}]$	$29(2)^{(b)}$		38(3)	[24]
$Q_s(12^+)$	$[\text{b}]$	0.49(3)	[28]	0.65(5)	[29]
$ V_{ZZ}(\text{PbRe}) $	$[10^{21} \text{ V/m}^2]$	2.42(23)	[25]	2.42(27)	[24]
<hr/>					
$ \nu_Q/\mu(11^-) $	$[\text{b}/\mu_N]$	25(3)	[25]	19(3)	[24]
<hr/>					
$\mu^{old}(11^-)$	$[\mu_N]$			+10.56(88)	[16]
$ \nu_Q^{old}(11^-) $	$[\text{MHz}]$	262(41)	[25]	199(32)	[24]
$Q_s^{old}(11^-)$	$[\text{b}]$	$(-4.48(86))$	[25]	$(-3.41(66))$	[24]
$\beta_2^{old}(11^-)$		$(-0.21(4))$	[25]	$(-0.156(28))$	[24]
<hr/>					
$\mu^{new}(11^-)$	$[\mu_N]$	+11.3(2)	[7]	+11.2(5) <sup>(c)</sup>	[16, 7]
$ \nu_Q^{new}(11^-) $	$[\text{MHz}]$	281(38)	this work	211(30)	this work
$Q_s^{new}(11^-)$	$[\text{b}]$	$(-4.8(7))$	this work	$(-3.6(6))$	this work
$\beta_2^{new}(11^-)$		$(-0.23(3))$	this work	$(-0.17(3))$	this work

<sup>(a)</sup> Weighted mean of the values in ref. [27].

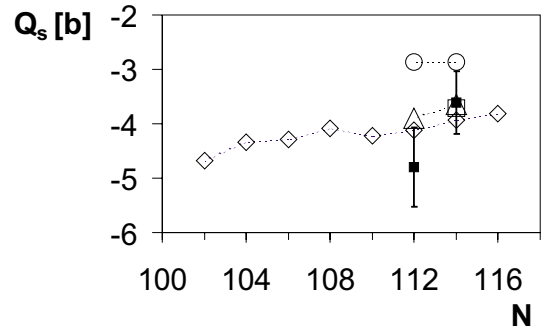
<sup>(b)</sup> The value slightly differs from the value in ref. [25] since it was deduced by using a slightly different magnetic moment (see also text).

<sup>(c)</sup> Weighted mean of the values in refs. [16] and [7].

First calculations on the deformation of the  $11^-$  isomer in  $^{196}\text{Pb}$  have been made by Bengtsson *et al.* [31]. They made use of a Woods-Saxon potential, the monopole pairing interaction, the standard Strutinsky renormalization and the Lipkin-Nogami treatment of the pairing problem. Similar values for the first excited  $0^+(\pi[505\ 9/2])^2$  state and the  $11^-(\pi[505\ 9/2]\ \pi[606\ 13/2])$  configuration in  $^{196}\text{Pb}$  have been found:  $\beta_2 \simeq -0.17$ . This calculated deformation is in perfect agreement with the deformation extracted from our experiments.

Recently, a systematic theoretical study of the deformation of the  $11^-$  isomers in the  $^{184-198}\text{Pb}$  and  $^{188-200}\text{Po}$  isotopes has been performed [32]. The calculations were performed in the framework of the Hartree-Fock-Bogoliubov method with a Skyrme interaction, a density dependent pairing force and the Lipkin-Nogami prescription of the pairing problem. Smirnova *et al.* found deformations ranging from  $\beta_2 = -0.15$  for  $^{198}\text{Pb}$  to  $\beta_2 = -0.19$  for  $^{184}\text{Pb}$ , also in fair agreement with our experimental values. The deformations will further increase towards midshell. The calculations show that the deformations of the  $\text{Pb}(11^-)$  isomers are significantly larger than the deformations of the corresponding  $\text{Po}$  isomers, varying from  $\beta_2 = -0.114$  for  $^{200}\text{Po}$  to  $\beta_2 = -0.117$  for  $^{196}\text{Po}$ . This effect is explained in terms of core polarization due to breaking of the  $Z = 82$  core in the  $\text{Pb}$  isotopes

A different approach is suggested by the particle-core coupling model [33]. In this model, the  $2p - 2h$  intruder states in  $^A\text{Pb}$  are described as a coupling of the two proton particles to a  $^{A-2}\text{Hg}$  core, whereas the two particles in  $^{A+2}\text{Po}$  are coupled to a  $^A\text{Pb}$  core. Despite the quite different approach the conclusions from the particle-vibration coupling calculations are similar to the ones ob-



**Fig. 2.** Comparison between the experimental (filled symbols) and theoretical (empty symbols) spectroscopic quadrupole moments of the  $11^-$  isomers in  $^A\text{Pb}$  as a function of the neutron number. The theoretical values include the results from the Hartree-Fock-Bogoliubov method [32] (diamonds), particle-core calculations [33] (triangles), TAC calculations [24] (circles) and the calculations of Bengtsson *et al.* (square) [31], where the spectroscopic quadrupole moment was obtained from the deformation by applying the standard formulae (see also text).

tained from the mean field calculations: the core polarization is much stronger in the  $\text{Pb}$  isomers (for  $^{196}\text{Pb}$  roughly 4 times larger than the single-particle contribution) than in the  $\text{Po}$  isomers (for  $^{198}\text{Po}$  roughly of the order of the single-particle contribution). This gives rise to theoretical spectroscopic quadrupole moments ranging from  $Q_s = -0.85 \text{ b}$  to  $Q_s = -1.83 \text{ b}$  for the  $^{210-198}\text{Po}$  chain and from  $Q_s = -3.64 \text{ b}$  to  $Q_s = -3.90 \text{ b}$  for the  $^{196,194}\text{Pb}$  isomers. The theoretical values for  $\text{Pb}$  are again in good agreement with the experimentally deduced spectroscopic quadrupole moments.

Tilted Axis Cranking (TAC) calculations result in somewhat lower values for the spectroscopic quadrupole moments and the deformations of the  $^{194,196}\text{Pb}(11^-)$  isomers:  $Q_s = -2.86$  b and  $\beta_2 = -0.127$  [24]. In this model the size of the deformation is controlled by the quadrupole-quadrupole coupling constant,  $\kappa$ , which was chosen such that the quadrupole moment of the the  $\nu(1i_{13/2}^{-2})$  isomer,  $Q_s(12^+) = 0.65$  b [29], is reproduced.

An overview of the several theoretical values in comparison with the experimental results is given in fig. 2.

## 4 Summary

We have put together and critically evaluated all experimental values which have been used in order to extract the quadrupole moments of the  $11^-$  isomers in the Pb-nuclei. This was necessary after a new measurement of the  $g$ -factors of these states [7]. The resulting values,  $Q_s(^{196}\text{Pb}, 11^-) = (-)3.6(6)$  b and  $Q_s(^{194}\text{Pb}, 11^-) = (-)4.8(7)$  b, are about 10% larger than the previously published values, though still within the formerly quoted experimental uncertainties. The relative uncertainties have been reduced and are now of the order of 15%. The new results have been compared with all available theoretical calculations and are in fair agreement with most of them. They confirm the moderate deformation of the  $11^-$  intruder isomers.

K.V. acknowledges financial support from the Flemish Science Foundation (FWO-Vlaanderen). D.L.B. was partially financially supported by the Bulgarian National Science Foundation under contract No. F-908. Support through the IAP P05/7 project from the OSTC-Belgium is acknowledged.

## References

1. K. Heyde *et al.*, Phys. Rep. **102**, 291 (1983), and references therein.
2. J. L. Wood *et al.*, Phys. Rep. **215**, 101 (1992), and references therein.
3. S. Åberg *et al.*, Annu. Rev. Nucl. Part. Sci. **40**, 439 (1990), and references therein.
4. R. V. F. Janssens *et al.*, Annu. Rev. Nucl. Part. Sci. **41**, 321 (1991), and references therein.
5. A. Kumar Jain *et al.*, Atomic Data Nucl. Data Tables **74**, 283 (2000), and references therein.
6. R. M. Clark *et al.*, Annu. Rev. Nucl. Part. Sci. **50**, 1 (2000), and references therein.
7. K. Vyvey *et al.*, Phys. Rev. C **69**, 064318 (2004).
8. S. Chmel *et al.*, Phys. Rev. Lett. **79**, 2002 (1997).
9. D. L. Balabanski *et al.*, Eur. Phys. J. A **20**, 191 (2004) and Phys. Rev. Lett. (submitted).
10. G. D. Dracoulis *et al.*, Phys. Rev. C **60**, 014303 (1999).
11. G. D. Dracoulis *et al.*, Phys. Lett. B **432**, 37 (1998).
12. J. M. Lagrange *et al.*, Nucl. Phys. A **530**, 437 (1991).
13. A. J. M. Plompen *et al.*, Nucl. Phys. A **562**, 61 (1993).
14. J. J. Van Ruyven *et al.*, Nucl. Phys. A **449**, 579 (1986).
15. R. M. Clark *et al.*, Nucl. Phys. A **562**, 121 (1993).
16. J. Penninga *et al.*, Nucl. Phys. A **471**, 535 (1987).
17. G. D. Dracoulis *et al.*, Phys. Rev. C **63**, 061302(R) (2001).
18. B. Fant *et al.*, J. Phys. G **17**, 319 (1991).
19. P. Van Duppen *et al.*, Phys. Rev. Lett. **52**, 1974 (1984).
20. P. Van Duppen *et al.*, Phys. Lett. B **154**, 354 (1985).
21. N. Bijnens *et al.*, Z. Phys. A **356**, 3 (1996).
22. A. N. Andreyev *et al.*, J. Phys. G **25**, 835 (1999).
23. A. N. Andreyev *et al.*, Nature **405**, 430 (2000).
24. K. Vyvey *et al.*, Phys. Rev. Lett. **88**, 102502 (2002).
25. K. Vyvey *et al.*, Phys. Rev. C **65**, 024320 (2002).
26. F. Hardeman *et al.*, Phys. Rev. C **43**, 130 (1991).
27. P. Raghavan *et al.*, At. Data Nucl. Data Tables **42**, 189 (1989), and references therein.
28. C. Stenzel *et al.*, Z. Phys. A **322**, 83 (1985).
29. S. Zywietz *et al.*, Hyperfine Interact. **9**, 109 (1981).
30. K. E. G. Löbner *et al.*, Nucl. Data Tables A **7**, 495 (1970).
31. R. Bengtsson and W. Nazarewicz, Z. Phys. A **334**, 269 (1989).
32. N. A. Smirnova *et al.*, Phys. Lett. B **569**, 151 (2003).
33. K. Vyvey *et al.*, Phys. Lett. B **538**, 33 (2002).